Accelerating MUS Extraction with Recursive Model Rotation

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Introduction

Minimal Unsatisfiability

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- $F = \{C_1, \ldots, C_6\} \in \text{UNSAT}$, but $\notin \text{MU}$.

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Applications of MUSes (in formal methods)

- Abstraction refinement frameworks.
- Decision procedures.
- Design debugging.

Computation of MUSes

- Based on iterative calls to SAT solver (not the only way, but currently the most effective): for each $C \in F$
  - if $F \setminus \{C\} \in$ UNSAT, then there is an MUS of F that does not contain $C \rightarrow$ remove C from F.
  - if $F \setminus \{C\} \in$ SAT (C is *necessary* for F), then C is in all MUSes of F $\rightarrow$ keep C.

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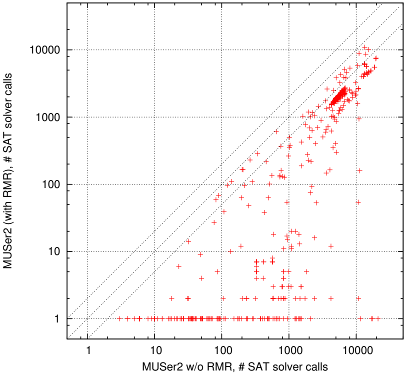
- On UNSAT outcomes – *clause set refinement*: remove C and all clauses outside the unsatisfiable core of $F \setminus \{C\}$. [Dershowitz et al'06]
- On SAT outcomes – *model rotation*: detect additional necessary clauses without SAT solver calls. [Marques-Silva&Lynce'11]

  Recursive model rotation (RMR) – very effective improvement of model rotation. [this paper]

Impact of RMR

- 500 benchmarks submitted to MUS track of SAT Competition 2011.
- Time limit 1200 sec, memory limit 4 GB.



- MUS computation without RMR (x-axis) vs with RMR (y-axis)
  - Left: number of SAT solver calls (on instances solved in both cases).

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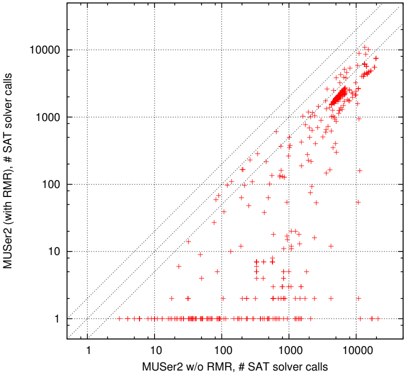

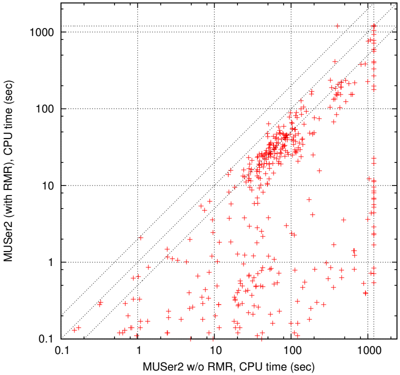

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  - Left: number of SAT solver calls (on instances solved in both cases).
  - Right: CPU time (sec).

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Use SAT solver to identify *necessary* (or, *transition*) clauses

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Deletion-based MUS Computation

Input : F — an unsatisfiable CNF formula

```
M ← F          // Inv: M is a superset of some MUS of F
foreach C ∈ F do
    if M \ {C} ∈ UNSAT then     // is C necessary for M ?
        // no - delete it
        M ← M \ {C}
    // yes - keep it
return M                   // Every C ∈ M is necessary for M
```

Example

$F = \{C_1, \ldots, C_6\}$

M (an overapproximation of some MUS of F):

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- Each clause in M costs ≤ 1 SAT solver call – model rotation.

Model Rotation [Marques-Silva&Lynce, SAT'11]

- **Fact:** C is necessary for F iff $F \in$ UNSAT and $\exists\tau$ such that $Unsat(F, \tau) = \{C\}$. τ is a *witness* (of necessity) for C.

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  - During MUS extraction: when $M \setminus \{C\} \in$ SAT, the assignment τ found by the SAT solver is a witness for C.
- *Model rotation:* given a witness τ for C, try to modify it into a witness τ' for another clause C'. How ?

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Flip x in τ': back to τ. C_3 is already known to be necessary.

Example

$F = \{C_1, \ldots, C_6\}$

M (the overapproximation of some MUS of F):

	$C_3 = x \vee \neg y$	$C_5 = y \vee z$
$C_2 = \neg x \vee y$	$C_4 = \neg x \vee \neg y$	$C_6 = y \vee \neg z$

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C_4 is necessary, without SAT solver call.

Recursive Model Rotation (RMR) [this paper]

- Simple idea: when model rotation stops, backtrack to a necessary clause detected earlier and flip another variable.

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- **Fact:** let τ be a witness for C in F, that is $\mathit{Unsat}(F, \tau) = \{C\}$. Then, the sets $\mathit{Unsat}(F, \tau|_{\neg x})$ for $x \in \mathit{Var}(C)$ are pairwise disjoint.
  - By flipping different variables we are likely to detect new necessary clauses.

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Tried all variables in C_4 — ~~stop~~ go back to C_3 and τ.

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C_4, C_5, C_6 are necessary, without SAT solver call.

Recursive Model Rotation (RMR)

Input: M — an over-approximation of an MUS
: C — a clause necessary for M
: τ — a witness for C (i.e. $Unsat(M, \tau) = \{C\}$)

foreach $x \in Var(C)$ **do**
$\tau' \leftarrow \tau|_{\neg x}$ `// flip` x
if $Unsat(M, \tau') = \{C'\}$ *and* C' *is not known to be necessary for* M **then**
mark C' as necessary
RMR(M,C',τ')

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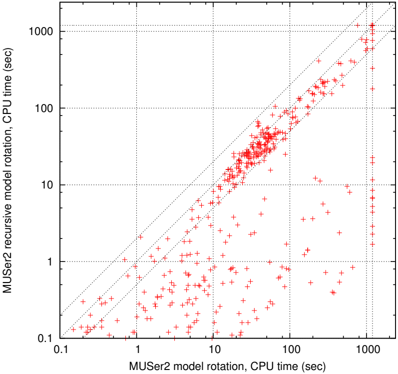
- The second condition of **if** keeps the number of the recursive calls linear in the size of computed MUS.

Recursive Model Rotation (RMR)

- 500 benchmarks submitted to MUS track of SAT Competition 2011.
- Time limit 1200 sec, memory limit 4 GB.



- Left: model rotation (x-axis) vs. RMR (y-axis), CPU time (sec).

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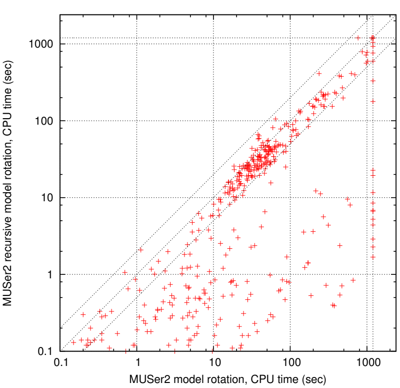
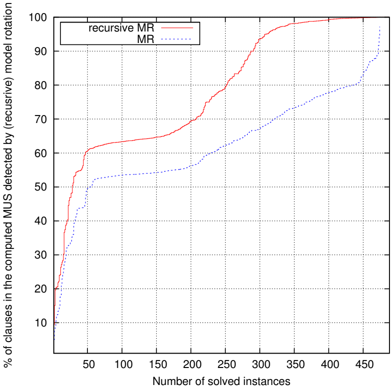
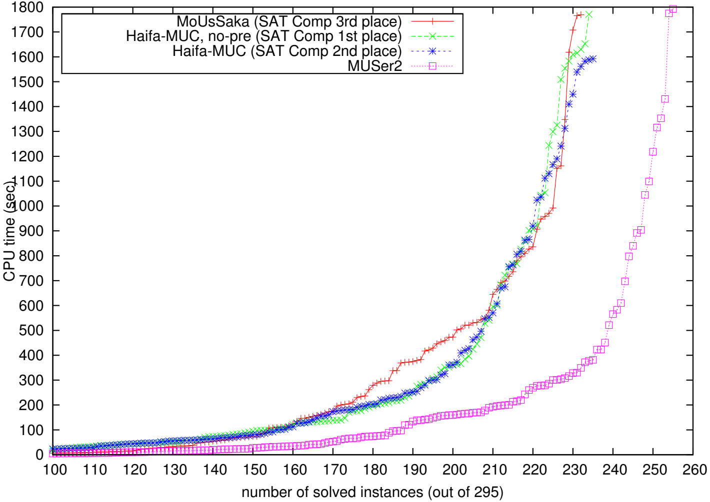




- Left: model rotation (x-axis) vs. RMR (y-axis), CPU time (sec).
- Right: % of clauses in the computed MUS detected by RMR (red) and by (non-recursive) model rotation (blue).

MUSer2 — MUS extractor with RMR

- 295 benchmarks used in the MUS track of SAT Competition 2011.
- Time limit 1800 sec, memory limit 4 GB.



Summary

- Recursive Model Rotation (RMR) — simple but powerful technique for acceleration of MUS extraction.
- Clause reordering (see the paper) — gives a slight performance edge.
- MUSer2 — state-of-the-art MUS extractor
  - Download at `http://logos.ucd.ie/wiki/doku.php?id=muser`

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Thank you for your attention !

Impact of RMR

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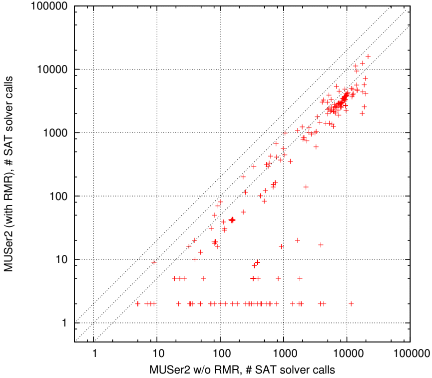

- MUS computation without RMR (x-axis) vs with RMR (y-axis)
  - Left: number of SAT solver calls (instances solved in both cases).

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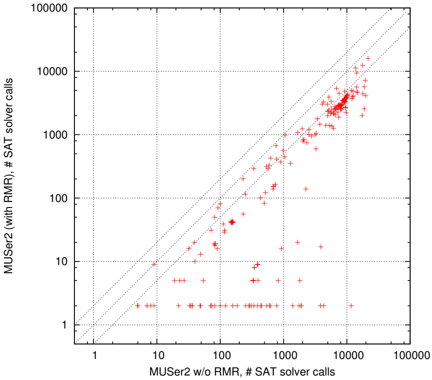

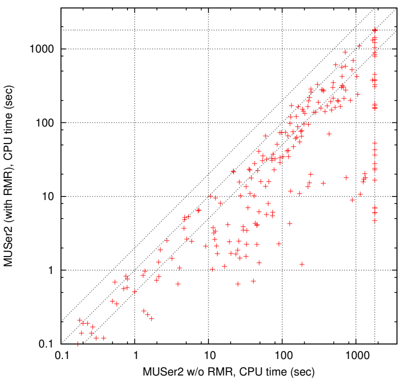

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  - Right: CPU time (sec).

Model Rotation [Marques-Silva&Lynce, SAT'11]

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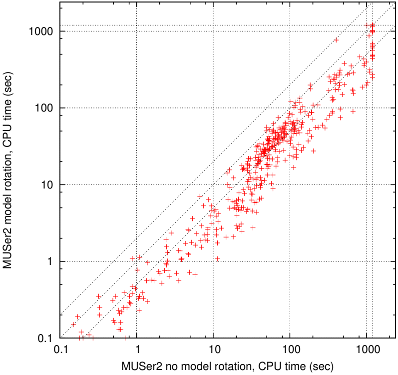

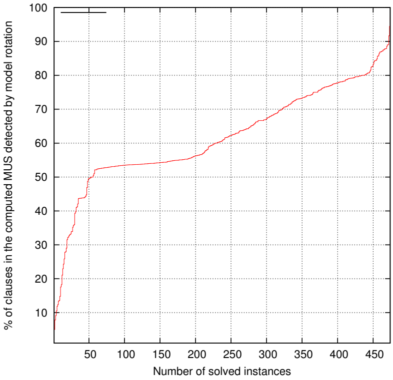

- Left: no model rotation (x-axis) vs. model rotation (y-axis).
- Right: % of clauses in computed MUS detected by model rotation.