An Incremental Approach to Model Checking Progress Properties

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Outline

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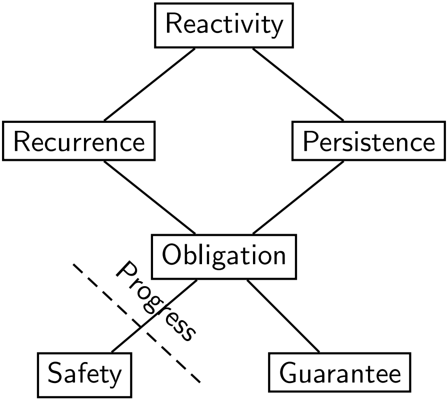
Property Classification



Linear Time Hierarchy

Safety: IC3
Progress: FAIR over IC3

Generalized Büchi Automata

- Given:
  - Fair Transition System (FTS) $\mathcal{S}$
  - LTL property P
- Compute **generalized Büchi automaton** $\mathcal{C} = \mathcal{A}_{\neg P} \parallel \mathcal{S}$.
- If $\mathcal{S}$ is finite state, nonemptiness of $\mathcal{C}$ corresponds to the existence of a **reachable fair cycle**, aka **lasso**.

Strongly Connected Components

- A lasso's cycle is contained in a **strongly connected component** (SCC) of the state graph
- A nonempty set of states is **SCC-closed** if every SCC is either contained in it or disjoint from it
- A partition of the states into SCC-closed sets is a coarser partition than the SCC partition; hence, . . .
- Every cycle of a graph is contained in some SCC-closed set

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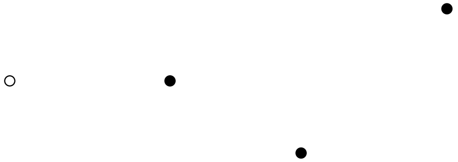
Reachable Fair Cycles

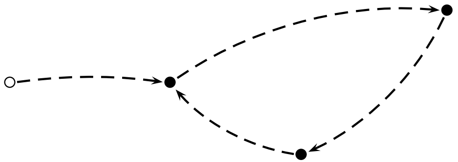
Reduce search for reachable fair cycle to a set of safety problems:

- Skeleton:



  States of skeleton together satisfy all fairness constraints.

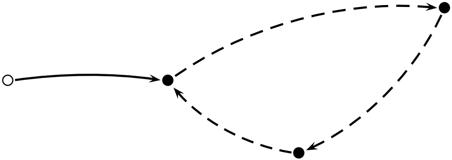
- Task: Connect states to form lasso.



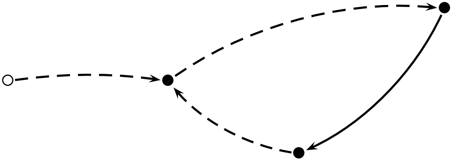
Reach Queries

Each connection task is a reach query.

- **Stem query**: Connect initial condition to a state:



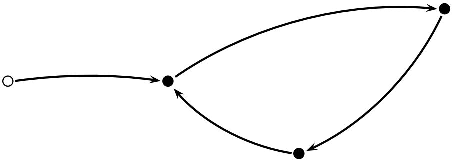
- **Cycle query**: Connect one state to another:



(To itself if skeleton has only one state.)

Witness to Nonemptiness

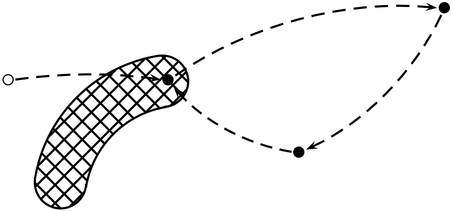
If all queries are answered positively:



Witness to nonemptiness of $\mathcal{C}$.

Global Reachability

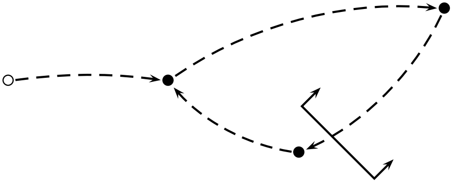
If a stem query is answered negatively: new **inductive** global reachability information.



- Constrains subsequent selection of skeletons.
- Constrains subsequent reach (stem and cycle) queries.
- Improve proof by strengthening (using ideas from IC3).

Barriers: Discovering SCC-Closed Sets

If a cycle query is answered negatively: new information about SCC structure of state graph.



- **Inductive** proof: "one-way barrier"
- Each "side" of the proof is SCC-closed.
- Constrains subsequent selections of skeletons: all states on one side.

Using Barriers for Generalization

- Can be used to constrain subsequent cycle queries.
  - Not necessary for completeness.
  - Can increase IC3's generalization power.
  - But can negatively impact SAT solver.
  - Must choose carefully which barriers to use.
- Improve proof by making smaller (using ideas from IC3).

Key Insights

- **Inductive assertions** describe **SCC-closed sets**.
- **Arena**: Set of states all on the same side of each barrier.
- Unlike previous symbolic methods:

  Barrier constraints on the transition relation *combined with the* **over-approximating nature of IC3** *enable the* **simultaneous (symbolic) consideration** *of all arenas.*

- A proof can provide information about many arenas even though the motivating skeleton comes from one arena.

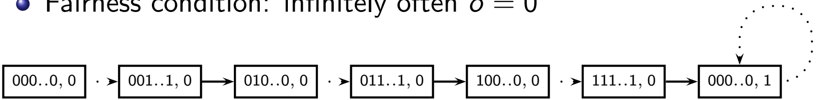
Methodological Parallels with IC3

	IC3	**FAIR**
Seed:	CTI	Skeleton
Lemma:	Inductive clause	Global reachability proof One-way barrier
	Relative to previously discovered lemmas.	
CEX:	CTI sequence	Connected skeleton
	Discovery guided by lemmas. Not minimal.	
Proof:	Inductive strengthening	All arenas skeleton-free
	Sufficient set of lemmas.	

Skeleton-Independent Proofs

Motivating example: *n*-bit counter

- Latches: $b_0, \ldots, b_{n-1}$ (least- to most-significant)
- Output: o switches to 1 and stays when all $b_i = 1$
- Initially: all 0
- Fairness condition: infinitely often $o = 0$

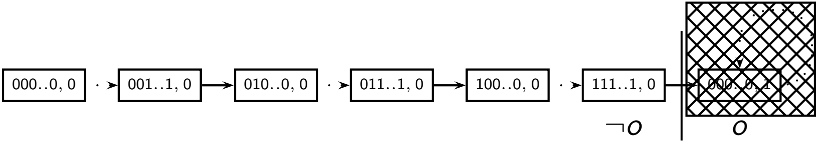


Unfair: after first rollover, henceforth $o = 1$.

Ideal Proof

First barrier: o

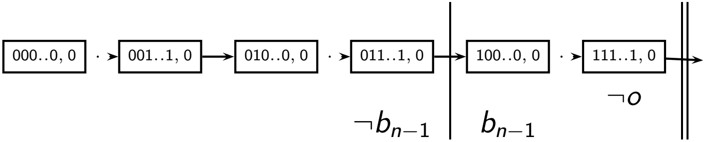
- Inductive because once $o = 1$, it stays 1
- No skeletons among o-states
- Constrain cycle queries: $\neg o \wedge \neg o'$



Ideal Proof

Second barrier: b_{n-1}

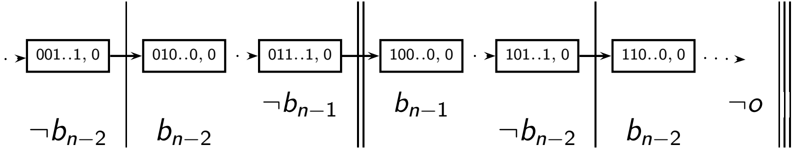
- Inductive relative to $\neg o$
- Once $b_{n-1} = 1$, it stays 1 in the $\neg o$-arena
- Both sides have skeletons
- Constrain cycle queries: $b_{n-1} \leftrightarrow b'_{n-1}$



Ideal Proof

Third barrier: b_{n-2}

- Inductive relative to previous barriers
- Once $b_{n-2} = 1$, it stays 1 in **every arena defined by the previous barriers**
- Both sides have skeletons in at least one arena
- Constrain cycle queries: $b_{n-2} \leftrightarrow b'_{n-2}$



And so on. Proof is linear in size of model.

Skeleton-Independent Proofs

- Only a lucky sequence of skeletons would yield ideal proof.
- Therefore: periodically test given predicates, such as single literals, to see if they are barriers (relative to current information).
- A predicate that is not an inductive barrier at one point can become inductive with new information.

Characteristics of FAIR

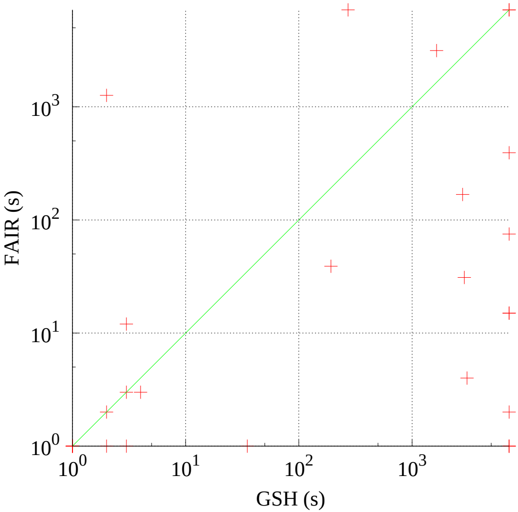
- Property directed (except skeleton-independent proofs)
- Relies on IC3, thus capitalizes on its strengths
- With IC3, approximating/abstracting
- Highly parallelizable even beyond IC3

Outline

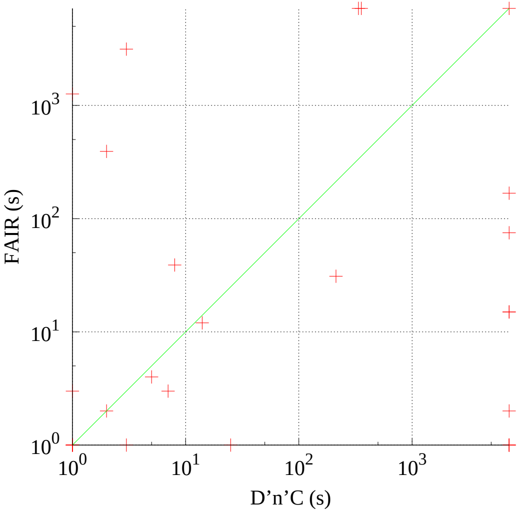
Experiments

- Evaluation on 30 models from 9 families
    - Contributed to the HWMCC11 benchmark set
    - Some from literature, most of which contrived
    - Most from VIS benchmark set
    - Number of fairness constraints ranges from 1 to 33
- Four different settings of FAIR considered
- Results compared to those of six other methods
    - Three BDD-based methods: GSH, Lockstep, D'n'C
    - Three variations of the liveness-to-safety scheme

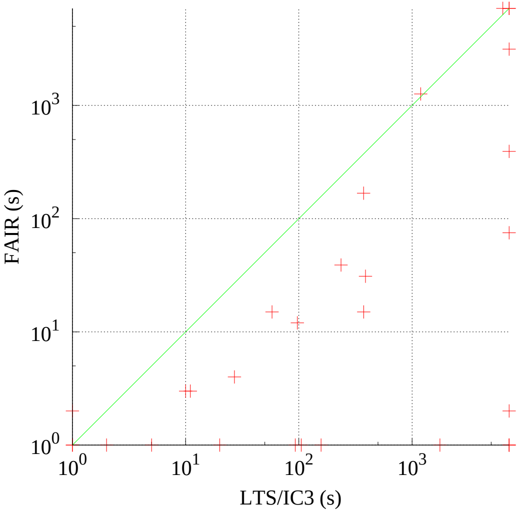
FAIR Compared to GSH



FAIR Compared to D'n'C



FAIR Compared to LTS/IC3



Results in Summary

- FAIR solved 27–28 problems out of 30 (depending on variation)
- GSH, D'n'C, LTS/IC3 solved 21 problems each
- LTS/ABC solved 20 problems
- Lockstep suffers when there are many SCCs (solved 12 problems)
- LTS/ITP solved 9 problems

Outline

Going Forward

- Selection of skeletons
- Proof improvement
- Deciding when to use a barrier to constrain cycle queries
- SAT solver: efficient handling of DNF
- SAT solver: highly incremental
- Distributed implementation
- Integrating BDDs

Conclusions

FAIR: a new approach to SAT-based LTL model checking

- In fact, to model checking all ω-regular properties
- Discovery of SCC-closed sets via safety queries
- One-way barriers: (relatively) inductive assertions
- Property-focused, approximating
- Not only uses IC3 but also follows its principles